Standard Random Variables

Notation: P{X} / P(X) , E[X] / E(X), assume ***independent*** *and identical distribution (iid).* Python: *np.random*

Memoryless $P(X > s + t | X > s) = P(X > t)$.

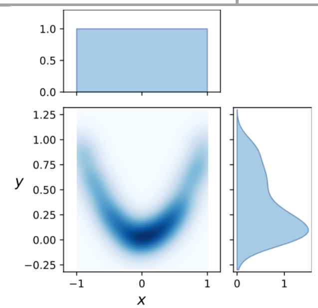
Discrete Distribution $X \in N$ Categorical $X \sim Cat([p_1, p_2, ..])$ →	**PMF** *Prob. Mass Function* *Valid* $i.\ \forall x_i.\ P\{X = x_i\} \geq 0$ $ii.\ \sum_{i=0}^{\infty} P\{X = x_i\} = 1$ (density sum to 1)	**CDF** *Cumulative Distribution* $F_X(x) = P\{X \leq \lfloor x \rfloor\}$, $1 - P\{X > x\}$, $P\{X = k\} = P\{X \leq k\} - P\{X \leq k-1\}$	$\boldsymbol{E[X]} =$ $\sum_{i=0}^{\infty} x_i P\{X = x_i\}$	$\boldsymbol{Var[X]} =$ $E[X^2] - E[X]^2$ *LOTUS*, $E[g(X)] =$ $\sum_x g(x) \boldsymbol{P}\{X = k\}$
Bernoulli trial $X \sim Bern(p)$	$P\{X\} = p,\ \ P\{\bar{X}\} = q$	$q = (1 - p)$	p	pq
Binomial *with replace* $X \sim Bin(n, p)$ #successes in n $Bern(p)$ trials $X \sim Bin(1, p)$, *(0-1) distribution* if *n=1*	$P\{X = k\} = \binom{n}{k} p^k q^{n-k}$ $P\{X = k\} = p^k q^{1-k}$	*Normal Approximation* *Poisson* $n \to \infty,\ p \to 0,\ \ \lambda = np$ *is moderate*	np	npq
Geometric / Negative Binomial $X \sim Geom(p),\ \ X \sim \boldsymbol{NegBin}(r,\ p)$ .... in n $Bern(p)$ trials until 1^{st}/r successes	$P\{X = k\} = q^k p, \quad k = \#failures$ $P\{X = k\} = \binom{\mathbf{n-1}}{\mathbf{r-1}} q^{k-1} p^r,\ k = \#trials$	$1 - q^{k+1},\ x > 0$ $[Exp(\lambda)]$ *Approximation*	$\frac{q}{p},\ \ \frac{1 \cdot \boldsymbol{r}}{p}$	$\frac{(1-p) \cdot \boldsymbol{r}}{p^2}$
Poisson $X \sim Pois(\lambda),\ \lambda = np > 0$ ***memoryless*** #events in a fixed interval of time t	$P\{X = k\} = \frac{e^{-\lambda} \lambda^k}{k!}$ $Pois(\lambda t)$ *given* $Exp(\lambda)$ *as waiting time interval*	*by def*	λ	λ
[Negative] HyperGeometric *no replace* $X \sim \boldsymbol{N}HGemo(w, b, n), total\ N = w + b$ #successes in n draws / until $\boldsymbol{n}$ **failures**	$P\{X = k\} = \frac{\binom{w}{k}\binom{b}{n-k}}{\binom{w+b}{n}}$	/	$np = n\frac{w}{N}$ $n\frac{w}{\boldsymbol{b+1}}$	$\frac{N-n}{N-1} npq$
Joint (X, Y)	$f_{X,Y}(x_i, y_j) = P_{X,Y}\left\{X = x_i,\ Y = y_j\right\}$ $= P_{ij}$	$F_{X,Y}(x, y) = \sum_{u=0}^{i} \sum_{v=0}^{j} P_{uv}$ $= P_{X,Y}\{X \leq x_i, Y \leq y_j\}$	*Valid* $\sum_{i=0}^{\infty} \sum_{j=0}^{\infty} P_{ij} = 1$	
Marginal (X) marginalize over another variable	$f_X(x_i) = P\{X = x_i\}$ $= \sum_y f_{X,Y}(x_i, y) = \sum_{j=0}^{\infty} P_{ij}$	$F_X(x_i) = P_X\{X \leq x_i\} = F(x_i,\ \infty)$ $= \sum_{u=0}^{i} f_X(x_u)$	$\forall i, j.\ P_{ij} \geq 0$	

X \ Y	1	2	3	
1	0.32	0.03	0.01	0.36
2	0.06	0.24	0.02	0.32
3	0.02	0.03	0.27	0.32
	0.40	0.30	0.30	1

Marginals for X $g(x) = \sum_y f(x, y)$

Marginals for Y → $h(y) = \sum_x f(x, y)$

$\sum_X \sum_Y f(x, y) = 1$



the joint density function

Continuous Distribution $X \in R$	**PDF** *Prob Density Function* Valid $i. \forall x.\ f_x(x) \geq 0$ $ii. \int_{-\infty}^{\infty} f_X(x)dx = 1$ (density sum to 1)	**CDF** *Cumulative Distribution* $F_X(x) = \int_{-\infty}^{x} f_X(t)dt$, *complement, LoTP* $f_X(x) = \frac{d}{dx} F_X(x)$	$\boldsymbol{E[X]} =$ $\int_{-\infty}^{\infty} x \cdot f(x)dx$	$\boldsymbol{Var[X]}$ LOTUS, $E[g(X)] =$ $\int_x g(x)f(x)dx$
Uniform $X \sim U(a,b) = e^{-\lambda Exp(\lambda)}$ a completely random point in $[low, high]$	$f_X(x) = \begin{cases} \frac{1}{b-a}, & x \in [a,b] \\ 0, & otherwise \end{cases}$	$F_X(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & x \in [a,b] \\ 1, & x > b \end{cases}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Exponential $X \sim Exp(\lambda)$, $rate\ \lambda = \frac{1}{\theta} > 0$ **memoryless** waiting time between 2 successive events	$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0, & otherwise \end{cases}$	$F_X(x) = \begin{cases} 1 - e^{-\lambda x}, & x > 0 \\ 0, & otherwise \end{cases}$	$\theta = \frac{1}{\lambda}$ *integrate by part*	$\theta^2 = \frac{1}{\lambda^2}$ */ tabular*
Pareto $X \sim Pareto(shape = \alpha) = x_m e^{Exp(\lambda)} / U^{-\frac{1}{\alpha}}$ cascade events, wealth, $x_m = 1$. **Loglinearity**	$f_X(x) = \begin{cases} \alpha x_m^{\alpha} x^{-(\alpha+1)}, & x \geq x_m \\ 0, & x < x_m \end{cases}$	$F_X(x) = \begin{cases} 1 - \left(\frac{x_m}{x}\right)^{\alpha}, & x \geq x_m \\ 0, & x < x_m \end{cases}$	$\begin{cases} \frac{\alpha x_m}{\alpha - 1}, & \alpha > 1 \\ \infty, & \alpha \leq 1 \end{cases}$	$\begin{cases} valid, & \alpha > 2 \\ \infty, & \alpha \leq 2 \end{cases}$
Normal / Gaussian $X \sim N(0,1)$ *Standard* $X \sim N(\mu, \sigma^2)$ $normal(loc = \mu,\ scale = \sigma)$	$f_X(x) = ce^{-\frac{x^2}{2}}$, $x \in R$, $c = \frac{1}{\sqrt{2\pi\sigma}}$ $f_X(x) = ce^{-\frac{(x-\mu)^2}{2\sigma^2}} = ce^{-\frac{(\frac{x-\mu}{\sigma})^2}{2}}$	$\Phi_X(x) = c\int_{-\infty}^{x} e^{-\frac{x^2}{2}} dx$ $\Phi_{\frac{X-\mu}{\sigma}}(z)$ CLT; $\Phi(z) = 1 - \Phi(-z)$	μ	σ^2
Beta $X \sim Beta(\alpha, \beta) = \frac{\Gamma(\alpha,1)}{\Gamma(a,1)+\Gamma(\beta+1)}$ As prior for Bayesian	$f_X(x) = \begin{cases} \frac{1}{B} x^{\alpha-1}(1-x)^{\beta-1}, & x \in [0,1] \\ 0, & otherwise \end{cases}$	$B = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)} = \frac{(\alpha-1)!(\beta-1)!}{(\alpha+\beta-1)!}$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$
Gamma $X \sim \Gamma(k,1)$, *shape k≻0* $X \sim \Gamma(k,\lambda)$, *scale=1/λ≻0* $Y \mid X \sim N(\mu, \frac{1}{\lambda X})$	$f_X(x) = \frac{1}{\Gamma(k)} x^{k-1} e^{-x}$, $x > 0$ $f_X(x) = \frac{1}{\Gamma(k)} \frac{(\lambda x)^k}{x} e^{-\lambda x}$, $x > 0$	$\Gamma(k) = (k-1)!$, $k \in N$ $\Gamma(k) = \int_0^{\infty} e^{-x} x^{k-1} dx$, $k \in R^+$ $\Gamma(1,\lambda) = Exp(\lambda), -\frac{1}{\lambda}\log(U)$	$\frac{k}{\lambda}$	$\frac{k}{\lambda^2}$
Joint (X, Y)	$f_{X,Y}(x,y) = \frac{\partial}{\partial x \partial y} F_{X,Y}(x,y)$	$F_{X,Y}(x,y) = \iint_B f_{X,Y}(x,y)dxdy$ $= \int_{-\infty}^{x}\int_{-\infty}^{y} f_{X,Y}(u,v)dudv$	Valid $F_{X,Y}(-\infty, \infty) = 1$	
Marginal (X)	$f_X(x) = \int_{y=-\infty}^{\infty} f_{X,Y}(x,y)dy$	$F_X(x) = P_X\{X \leq x\} = F_{X,Y}(x, \infty)$ $= \int_{-\infty}^{x} f_X(u)du$	$\forall x, y.\ f_{X,Y}(x,y) \geq 0$	